B.Sc Part I (Honrs)

Example of Fundamental Concepts.

Question:- The equation $x^3-5x^2-4x+20=0$ has two roots whose difference is 3. Find ~~the~~ all the roots of the equation.

Ans:- Let α, β, and γ are three roots of the given equation

$$f(x)=x^3-5x^2-4x+20=0 \quad \text{---(1)}$$

From equation. $\beta-\alpha=3 \Rightarrow \beta=\alpha+3$

As β is one of the roots of $f(x)$

$$\therefore f(\beta)=0 \Rightarrow f(\alpha+3)=0$$

By(1) $f(\alpha+3)=(\alpha+3)^3-5(\alpha+3)^2-4(\alpha+3)+20=0$

$\Rightarrow \alpha^3+27+9\alpha^2+27\alpha-5(\alpha^2+9+6\alpha)-4\alpha-12+20$

$\Rightarrow \alpha^3+4\alpha^2+27+27\alpha-45-30\alpha+4\alpha+8=0$

$\Rightarrow \alpha^3+4\alpha^2-7\alpha-10=0 \quad \text{---(2)}$

Since α is the root of $f(x)$

$$\therefore f(\alpha)=\alpha^3-5\alpha^2-4\alpha+20=0 \quad \text{---(3)}$$

Now to find the common factor (HCF) between (2) and (3) we use following division.

$$(\alpha^3+4\alpha^2-7\alpha-10)\ \alpha^3-5\alpha^2-4\alpha+20\ (1$$

$$\underline{-\alpha^3 \pm 4\alpha^2 \mp 7\alpha \mp 10}$$

$$-3 \mid -9\alpha^2+3\alpha+30$$

$$3\alpha^2-\alpha-10)\ \alpha^3+4\alpha^2-7\alpha-10\ (\alpha+13$$

$$\underline{\qquad\qquad \times 3}$$

$3\alpha^3+12\alpha^2-21\alpha-30$

$3\alpha^3 \mp \alpha^2 \mp 10\alpha$

$13\alpha^2-11\alpha-30$

$\times 3$

$39\alpha^2-33\alpha-90$

$-39\alpha^2 \mp 13\alpha \mp 130$

$-20 \mid -20\alpha+40$

$\alpha-2$

(2) $\alpha-2)\,3\alpha^2-\alpha-10\,(3\alpha+5$

$-3\alpha^2-6\alpha$

$- \quad +$

$5\alpha-10$

$5\alpha-10$

$\times \quad \times$ Here HCF $=\alpha-2$

Thus the Common root of (2) and (3) is $\alpha=2$

By (1) $\beta=\alpha+3=2+3=5$

Hence $(x-2)$, $(x-5)$ are factors of $f(x)$.

For 3rd root divide $f(x)$ by $(x-2)(x-5)$ and we get $x=-2$

The roots of given equation are $-2, 2$ and 5.

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28.04.2021